Localized Structures of Electromagnetic Waves in Hot Electron-Positron Plasma

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ABSTRACT

The dynamics of relativistically strong electromagnetic (EM) wave propagation in hot electron-positron plasma is investigated. The possibility of finding localized stationary structures of EM waves is explored. It is shown that under certain conditions the EM wave forms a stable localized soliton-like structures where plasma is completely expelled from the region of EM field location.

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During the last few years a considerable amount of work has been devoted to the analysis of nonlinear electromagnetic (EM) wave propagation in electron-positron (e-p) plasmas [1]. Electron- positron pairs are thought to be a major constituent of the plasma emanating both from the pulsars and from inner region of the accretion disks surrounding the central black holes in the active galactic nuclei (AGN) [2]. The process of e-p pair creation occurs in relativistic plasma at high temperatures, when the temperature of the plasma becomes of the order of, or larger than, the rest energy of electrons. Such relativistic plasmas have presumably appeared in the early universe [3]. Intense relativistic e-p plasmas could also exist in the vicinity of cosmic defects like superconducting cosmic strings [4]. Collective processes in e-p plasmas are of considerable interest. The processes of wave self-modulation of EM waves and soliton formation have attracted a great deal of attention. Stable localized solution may be a potential mechanism for the production of micro-pulses in AGN and pulsars [5]. In the early universe stable localized EM waves could create inhomogeneities necessary to understand the observed structure of the visible universe.

In the recent paper of Berezhiani and Mahajan [6] the nonlinear propagation of relativistically strong EM radiation in a hot e-p plasma has been considered. It has been shown that e-p plasma supports the propagation of nondiffracting and nondispersive EM pulses (light bullets) with large density bunching. However, the authors concentrated in the case of transparent plasma and consequently the group velocity of the pulses is close to the ve-

locity of light c. In the present paper we consider the propagation of strong EM radiation in a hot unmagnetized e-p plasma aiming to find the localized stationary soliton-type solutions.

We start from Maxwell equations to describe the EM wave propagation in an e-p plasma, expressing the fields by the vector and scalar potentials, i.e.

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - \nabla\varphi \ , \qquad \mathbf{B} = \nabla \times \mathbf{A} \tag{1}$$

where the coulomb gauge $\nabla \cdot \mathbf{A} = 0$ is fulfilled. Accordingly the field equations take the form:

$$\frac{\partial^2 \mathbf{A}}{\partial t^2} - c^2 \Delta \mathbf{A} + c\frac{\partial}{\partial t}(\nabla\varphi) - 4\pi c \mathbf{J} = 0 \tag{2}$$

and

$$\Delta\varphi = -4\pi\rho \tag{3}$$

Here, ρ and $\mathbf{J}$ are the charge and current densities given by

$$\rho = \sum_\alpha e_\alpha n_\alpha, \qquad \mathbf{J} = \sum_\alpha e_\alpha n_\alpha \mathbf{u}_\alpha \tag{4}$$

where α indicates the particle species α $(= e, p$ for electrons and positrons, respectively); e_α and n_α are the charge and density of the corresponding particle α. We consider the case in which the equilibrium state is characterized by $n_{0e} = n_{0p} = n_0$, where $n_{0\alpha}$ is the equilibrium density of the particle α.

Before writing the hydrodynamic equations of relativistic plasma it is necessary to define what relativistic means. In fact, we have two types of

relativistic regimes in plasma: in a strong EM field the plasma particles may obtain relativistic velocities. In space, the EM radiation of objects (nuclei of galaxies, radio-galaxies, quasars, pulsars, etc.) may serve as a source of such strong fields. The case when the thermal energy of the plasma particles is of the order of, or larger than, the energy at rest, it is the other type of relativistic regime. In this case the thermal velocities of the particles become of the order of the light speed. Certainly, in both cases the decisive role belongs to relativistic effects in plasma, but the character of its manifestation is different. Both relativistic effects can play an important role in the e-p plasma. Let us assume that the velocity distribution of the particles of species α is locally a relativistic Maxwellian. Then, according to Ref.[6, 7], the set of relativistic hydrodynamic equations of motion can be written as:

$$\frac{d}{dt}(m_{0\alpha}G_\alpha\gamma_\alpha c^2) - \frac{1}{n_\alpha}\frac{\partial}{\partial t}P_\alpha = e_\alpha \mathbf{u}_\alpha \mathbf{E} \tag{5}$$

$$\frac{d}{dt}(\mathbf{p}_\alpha G_\alpha) + \frac{1}{n_\alpha}\nabla P_\alpha = e_\alpha \mathbf{E} + \frac{e_\alpha}{c}(\mathbf{u}_\alpha \times \mathbf{B}) \tag{6}$$

The continuity equation for the particle α is

$$\frac{\partial n_\alpha}{\partial t} + \nabla(n_\alpha \mathbf{u}_\alpha) = 0 \tag{7}$$

Here $\mathbf{p}_\alpha = \gamma_\alpha m_{0\alpha}\mathbf{u}_\alpha$ is the hydrodynamic momentum, $P_\alpha = n_\alpha T_\alpha/\gamma_\alpha$ is the relativistic particle pressure, $\mathbf{u}_\alpha$ is the hydrodynamic velocity of the fluid, $\gamma_\alpha = (1 - u_\alpha^2/c^2)^{-1/2}$ is the relativistic factor, $m_{0\alpha}$ and T_α are the particles invariant rest mass, and temperature respectively, $d_\alpha/dt = \partial/\partial t + \mathbf{u}_\alpha\nabla$ is the comoving derivative. The role of the particle-mass is now played by

the quantity $M_{eff} = m_{0\alpha} G_\alpha(z_\alpha)$, where $G_\alpha(z_\alpha) = K_3(z_\alpha)/K_2(z_\alpha)$. Here $K_3(z_\alpha)$ and $K_2(z_\alpha)$ are respectively the McDonald functions of the second and third order $(z_\alpha = m_{0\alpha}c^2/T_\alpha)$. The effective mass of the particles M_{eff} depends on the temperature. For nonrelativistic temperatures $(T_\alpha \ll m_{0\alpha}c^2)$ $M_{eff} = m_{0\alpha}(1 + 5T_\alpha/2m_{0\alpha}c^2)$, while for ultrarelativistic high temperatures $(T_\alpha \gg m_{0\alpha}c^2)$ the effective mass becomes $M_{eff} = 4T_\alpha/c^2$, and the fluid inertia is primary provided by random thermal motion of the particles. In this case, the rest-mass is negligibly small and the e-p gas behaves like photons. Using simple manipulation, from the Eqs. (5) and (6), we obtain the adiabatic equation which reads

$$\frac{n_\alpha z_\alpha}{\gamma_\alpha K_2(z_\alpha)} exp(-z_\alpha G_\alpha(z_\alpha)) = const. \tag{8}$$

In the nonrelativistic limit $(z_\alpha \gg 1)$, Eq. (8) yields the result for a mono atomic ideal gas $(n_\alpha/(\gamma_\alpha T_\alpha^{3/2}) = const.)$, and in the ultrarelativistic limit $(z_\alpha \ll 1)$ we have the adiabatic law for the "photon" gas $(n_\alpha/(\gamma_\alpha T_\alpha^3) = const.)$.

We are looking for a localized one-dimensional solution of this system of equations for a circularly polarized EM wave, where the vector potential $\mathbf{A}$ can be expressed as:

$$\mathbf{A}_\perp = \frac{1}{2}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})A(z,t)exp(-i\omega_0 t) + c.c \tag{9}$$

where $A(z,t)$ is a slowly varying function of t, ω_0 is mean frequency , $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are the unit vectors. The transverse component of equation of motion (6) is immediately integrated to give (for details see Ref. [6]):

$$\mathbf{p}_{\alpha\perp} G_\alpha = -\frac{e_\alpha}{c}\mathbf{A}_\perp \tag{10}$$

where the constant of integration is set equal to zero, since the particle hydrodynamic momentum is assumed to be zero at infinity where the fields vanish.

Before writing the equations for the longitudinal motion we would like to mention that this motion of the plasma is driven by the ponderomotive pressure ($\sim p_{\alpha\perp}^2$) of high-frequency EM fields and it does not depend on the particle charge sign. In what follows we assume that in equilibrium the temperatures of electrons and positrons are equal, i.e. $T_{0e} = T_{0p} = T_0$. Since the effective mass of the electrons and positrons are equal ($G_e = G_p = G$), the radiation pressure gives equal longitudinal momenta to both the electrons and positrons ($p_{ez} = p_{pz} = p_z$) and affects concentration without producing the charge separation. Consequently $n_e = n_p = n$ and $\phi = 0$. It is also evident that due to symmetry between electron and positron fluids their temperatures remain equal ($T_e = T_p = T$).

It is now convenient to introduce the following dimensionless quantities:

$$\mathbf{p}_\alpha = \frac{\mathbf{p}_\alpha}{m_{0e}}, \quad n = \frac{n}{n_0}, \quad T = \frac{T}{m_{0e}c^2}, \quad \mathbf{A} = \frac{|e|\mathbf{A}}{m_{0e}c^2}, \quad \mathbf{r} = \frac{\omega_e}{c}\mathbf{r}, \quad t = \omega_e t \tag{11}$$

where $\omega_e = (4\pi e^2 n_0/m_{0e})^{1/2}$ is the electron Langmuir frequency.

The longitudinal motion of the plasma is determined entirely by the set consisting of the z component of the equation of motion (6),

$$\left(\frac{\partial}{\partial t}+u_z\frac{\partial}{\partial z}\right)Gp_z+\frac{1}{n}\frac{\partial}{\partial z}\frac{nT}{\gamma}=-\frac{1}{2\gamma G}\frac{\partial|A|^2}{\partial z} \tag{12}$$

and the "energy" conservation equation (5),

$$\left(\frac{\partial}{\partial t}+u_z\frac{\partial}{\partial z}\right)G\gamma-\frac{1}{n}\frac{\partial}{\partial t}\frac{nT}{\gamma}=\frac{1}{2\gamma G}\frac{\partial|A|^2}{\partial t} \tag{13}$$

where $u_z = p_z/\gamma$. The relativistic factor γ does not depend on the "fast" time (ω_0^{-1}) and can be written as:

$$\gamma=\left[1+\frac{|A|^2}{G^2}+p_z^2\right]^{1/2} \tag{14}$$

Substituting (9) and (10) into Eq.(2), then for a slowly varying amplitude of EM wave $A(z,t)$ we obtain the following equation:

$$2i\omega_0\frac{\partial A}{\partial t}+\frac{\partial^2 A}{\partial z^2}+\Delta\cdot A+2fA=0 \tag{15}$$

where

$$f=1-\frac{nG_0(T_0)}{\gamma G(T)} \tag{16}$$

and $\Delta = \omega_0^2 - 2$. For convenience we redefined the electron rest mass in Eq.(11) as $m_{0e} \rightarrow m_{0e}G_0(T_0)$. In dimensional units $\Delta \sim \omega_0^2 - 2\omega_e^2$, where $\omega_e = (4\pi e^2 n_0/m_{0e}G_0(T_0))^{1/2}$.

We are looking for the stationary localized solutions (vanishing at infinity) described by Eqs. (8), (12)-(16). Assuming that $|A|$ depends only on the special coordinate z, and integrating Eqs. (12) and (13), we get the following integral of motion

$$G(T)\gamma = G_0(T_0) \tag{17}$$

where $\gamma = (1 + |A|^2/G^2)^{1/2}$ $(p_z = 0)$. From Eq. (17) we get:

$$G = G_0 \left(1 - \frac{|A|^2}{G_0^2}\right)^{1/2} \tag{18}$$

It follows from Eq.(18) that the present hydrodynamical theory, which describes the nonlinear waves in e-p plasma, is valid for $|A|^2_{max}/G_0^2 \leq 1$. When the latter is violated, then the electromagnetic waves are overturned and they will cause multi-stream motion of the plasma. In such a situation, one must resort to kinetic description for studying the nonlinear wave motion. This investigation is, however, beyond the scope of the present paper.

Using Eqs. (17)-(18) we get $f = 1 - n$. Now we should obtain a relationship between n and $|A|^2$. To this end, one can use the adiabatic equation (8). Unfortunately it is impossible to solve this problem analytically for the arbitrary temperatures. In the nonrelativistic case $(T, T_0 \ll 1)$ Eq. (8) gives $n = \gamma(T/T_0)^{3/2}$ and using Eqs. (17)-(18) along with the asymptotic expression for G $(= 1 + \frac{5}{2}T)$ for the plasma density we obtain:

$$n = \left(1 - \frac{|A|^2}{5T_0}\right)^{3/2} \tag{19}$$

In the ultrarelativistic case $(T, T_0 \gg 1)$ from Eq. (8) we have $n = \gamma(T/T_0)^3$ and using the asymptotic expression for G $(= 4T)$ we get:

$$n = 1 - \frac{|A|^2}{16T_0^2} \tag{20}$$

The expressions (19) and (20) show that the total density of plasma can become zero at $|A|^2 = |A|^2_{cr}$ (where $|A|^2_{cr} = 5T_0$ for the nonrelativistic case and $|A|^2_{cr} = 16T_0^2$ for the ultra-relativistic case). This phenomenon can be called "electron-positron cavitation" and for the ultrarelativistic case has been discovered in Ref. [8]. It should be noted that the upper bound limitation of the amplitude of the vector potential is caused by the fact that we consider here the stationary case, and consequently the inertial terms in Eqs.(12)-(13) have been neglected. In the case when $|A|^2 > |A|^2_{cr}$, a gas-dynamic pressure force cannot compensate the ponderomotive one and a stationary distribution of fields does not exist.

In the ultrarelativistic case, $f = |A|^2/16T_0^2$ and Eq. (15) takes the form of the well known nonlinear Schrödinger equation. The stationary soliton solution of this equation (which corresponds to nonlinear frequency shift $\Delta = -A_m^2/16T_0^2$) is

$$A = A_m sech\left(\frac{A_m}{4T_0} z\right) \tag{21}$$

were A_m is amplitude of soliton. Note that amplitude of the soliton A_m can be relativistically strong ($A_m \gg 1$). The only restriction is that $A_m \leq A_{cr} = 4T_0$. If $A_m \to A_{cr}$ then the cavitation of plasma occurs and all particles are rejected from the central part of soliton.

Now let us consider the nonrelativistic case. Using Eq. (19) f can be written as

$$f = 1 - \left(1 - \frac{|A|^2}{5T_0}\right)^{3/2} \tag{22}$$

substituting Eq.(22) into Eq.(15) we get the nonlinear Schrödinger equation with a saturating nonlinearity. For the stationary soliton solution we should

solve to following equation:

$$\frac{d^2E}{dz^2} - \lambda^2 E + 2E[1-(1-E^2)^{3/2}] = 0 \tag{23}$$

where $E = |A|/(5T_0)^{1/2} \leq 1$ and $\Delta = -\lambda^2$. The Eq.(23) has a soliton solution provided that nonlinear frequency shift satisfies the following "dispersion" relation:

$$\lambda^2 = 2 - \frac{4}{5}\frac{1}{E_m^2}\left(1-(1-E_m^2)^{5/2}\right) \tag{24}$$

where E_m is the amplitude of the soliton. One can see that λ^2 monotonically grows with E_m^2 and when $E_m^2 \to 1$ (which corresponds to cavitation) obtains its maximal allowed value $\lambda_{max}^2 = 1.2$. Unfortunately the general analytical solutions of Eq.(23) can not be expressed in terms of elementary functions. We would like to mention that in the case of small amplitude solitons ($E_m \ll 1$) the soliton represents a soliton solution of the cubic nonlinear Schrödinger equation and can be written as:

$$E = E_m sech\left[\left(\frac{3}{2}\right)^{1/2} E_m z\right] \tag{25}$$

If $E_m \to 1$ ($\lambda^2 \to 1.2$) the top part of the soliton is well described by a *cosine* function and can be approximated as

$$E = E_m cos[(2-\lambda^2)^{1/2} z] \tag{26}$$

The general shape of the soliton is displayed in Fig.1 where $E_m = 0.99$ ($\lambda^2 = 1.19$). Dashed line corresponds to analytical approximation given by Eq.(26).

Using the well-known stability criterion of Vakhitov and Kolokolov [9], it can be shown that the above described soliton-type solution is stable against small perturbations.

In conclusion, we have considered the possibility of high-frequency EM wave localization in hot unmagnetized electron-positron plasmas. In our analysis, we included not only the relativistic effects in the hydrodynamic motion of the plasma, but also the effects which result from the relativistic electron velocity distribution. We have shown that in such plasmas it is possible to have localized stable soliton-like structures. It is also shown that cavitation of plasma can occur both in the nonrelativistic and ultrarelativistic cases. The present result should be useful for the understanding of the nonlinear photon motion in cosmical plasmas such as those found in the early universe and AGN.

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References

[1] C.F. Kennel and R. Pellat, J. Plasma Phys. **15**, 335 (1976); J.N. Leboeuf, M. Ashour-Abdalla, T. Tajima, C.F. Kennel, F. Coroniti, and J.M. Dawson, Phys. Rev. A**25**, 1023 (1982); M.E. Gedalin, J.G. Lominadze, L. Stenflo, and V.N. Tsitovich, Astrophys. Space Sci. **108**, 393 (1985); P.K. Shukla, N.N. Rao, M.Y. Yu, and N.L. Tsintsadze, Phys. Rep. **135**, 1 (1986); V.I. Berezhiani, V. Skarka, and S.M. Mahajan, Phys. Rev. E**48**, R3252 (1993).

[2] F.C. Michel, Rev. Mod. Phys. **54**, 1 (1982); M.C. Begelman, R.D. Blandford, and M.D. Rees, Rev. Mod. Phys. **56**, 255 (1984).

[3] K. Holcomb and T. Tajima, Phys. Rev. D**40**, 3909 (1989); T. Tajima and T.Taniuti, Phys. Rev. A**42**, 3587 (1990); P.K. Shukla, N.L. Tsintsadze, and L.N. Tsintsadze, Phys. Fluids B**5**, 233 (1993).

[4] E. Witten, Nucl. Phys. B**249**, 557 (1985); J.P. Ostriker, C. Thompson, and E. Witten, Phys. Lett. B**180**, 231 (1986).

[5] A.C.L. Chian and C.F. Kennel, Astrophys. Space Sci. **97**, 3 (1983); A.B. Mikhailovskii, O.G. Onishchenko, and E.G. Tatarinov, Plasma Phys. Contr. Fusion, **27**, 539 (1986); R.T. Gangadhara, V. Krishan, and P.K. Shukla, Mon. Not. R. astr. Soc. **258**, 616 (1992).

[6] V.I. Berezhiani and S.M. Mahajan, Phys. Rev. Lett. **73**, 1110 (1994); V.I. Berezhiani and S.M. Mahajan, Phys. Rev E**52**, 1968 (1995).

[7] D.I. Dzhavakhishvili and N.L. Tsintsadze, Sov. Phys. JETP **37**, 666 (1973); S.V. Kuznetsov, Sov. J. Plasma Phys. **8**, 199 (1982).

[8] L.N. Tsintsadze, Phys. Scr. **50**, 413 (1994).

[9] J. Juul Rasmussen and K. Rypdal, Phys. Scr. **33**, 481 (1986).

Figure Captions

Fig.1 Solution E as a function of the space coordinate z, ($E_m = 0.99$). The dashed line corresponds to analytical approximation given by Eq.(26).